Grade 7/8 Math Circles
February 27/28, March 1/2 2022,
BCC and Gauss Prep

Beaver Computing Challenge

The Beaver Computing Challenge (BCC) is an online problem-solving contest with a focus on computational and logical thinking. No prior coding experience is required. The questions are inspired by topics in computer science but students only require the concepts taught in the mathematics curriculum common to all provinces.

Students in grade 8 or below can write the Grade 7/8 BCC. The Grade 7/8 BCC consists of 15 multiple choice questions divided into 3 parts with 90 marks total: 5 questions in Part A worth 8 marks each, 5 questions in Part B worth 6 marks each, and 5 questions in Part C worth 4 marks each. For a maximum of 6 questions, it is possible to receive 2 marks for every unanswered question. Students are given exactly 45 minutes to answer the questions. Some calculators are permitted.

Each question on the BCC is given by a story and a question. The story provides the background information required to solve the question.

More information on the BCC: https://cemc.uwaterloo.ca/contests/bcc.html Past contests/solutions: https://www.cemc.uwaterloo.ca/contests/past_contests.html#bcc

Strategies

Solving the BCC problems requires computational and logical thinking. Listed here are a few strategies to approach these types of problems.

- First read the story, then the question, then reread the story. This will help with finding the details needed to solve the question as well as understanding what the question is asking.
- Underline or write down the important information in the story and question.
- If the question is long and/or challenging, split it into pieces or steps. Focus on one step at a time, then connect them all together at the end.
- Make a chart or diagram to help organize what is given in the story. Or create another image

that will help to visualize what is happening in the problem.

- Rule out answers that are impossible or that you can show aren't the solution. The problems are all multiple choice and will have 4 options, so ruling out a couple incorrect answers can help with deciding on the correct answer. When in doubt, make a logical guess.
- Have fun writing the contest! This contest is meant to be an enjoyable experience that will motivate your interest in math and computer science. The BCC emphasizes participation rather than competition, so be proud of trying.
- Following the end of the contest window, solutions will be posted on the website at this link: https://www.cemc.uwaterloo.ca/contests/past_contests.html#bcc

Past Contest Problems

Toy Storage

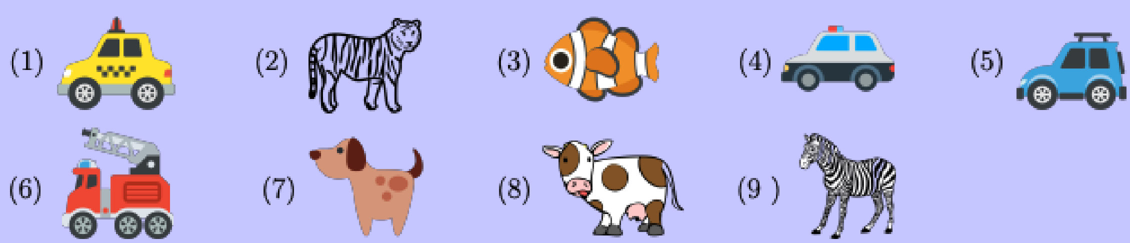
Tom has two types of toys: animal toys and vehicle toys. Tom fills three boxes by putting three toys in each box. As long as there is room, he puts

- vehicles into box A,
- animals with striped bodies into box B, and
- animals with spotted bodies into Box C.

However,

- Anytime he tries to put a toy into box A and it is full, he then tries to put the toy into box B.
- Anytime he tries to put a toy into box B and it is full, he then tries to put the toy into box C.
- Anytime he tries to put a toy into box C and it is full, he then tries to put the toy into box A.

Tom puts the following nine toys into boxes in the following order:



Where does Tom put the dog and zebra?

A. Tom puts the dog in box C, and the zebra in box B.

B. Tom puts both in box A.

C. Tom puts both in box B.

D. Tom puts both in box C.

Solution
For toys (1) to (5), we can follow the rules and put them into their corresponding boxes.
When Tom tries to put the firetruck into box A, he realizes that it is full (because it has the taxi, the policecar and the car). He then puts it into box B which still has room for one more toy. At this point, box B has the tiger, the clownfish and the firetruck.
Tom can easily place the dog and the cow into box C as there is still room.
When Tom tries to put the zebra into box B, he realizes that it is full, so he places the zebra into box C.
So the answer is (D), Tom puts both the dog and the zebra into box C.

Swapping Dogs

Two types of dogs are standing as shown below.

A *swap* occurs when two dogs that are beside each other exchange positions. After some swaps, the three large dogs end up in three consecutive positions.

What is the fewest number of swaps that could have occured?

A. 5

B. 6

C. 7

D. 8

Solution
The answer is (B), the fewest number of swaps that could have occured is 6.
The three large dogs can end up in three consecutive positions by doing the following:

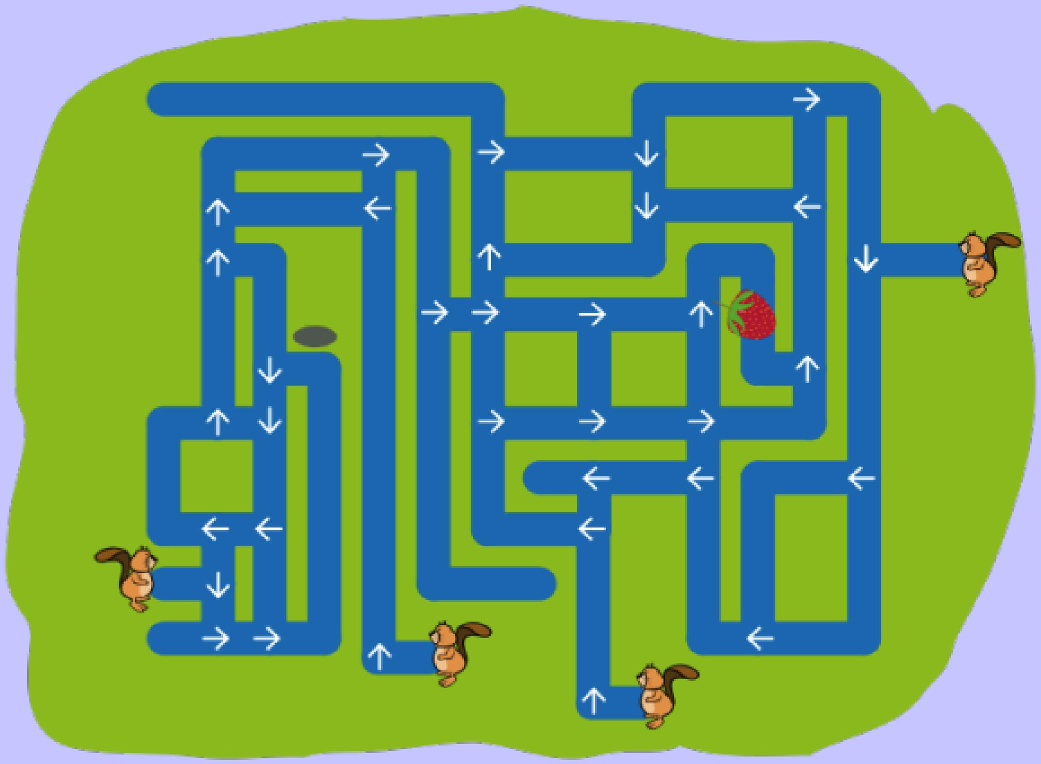
1. Swapping the large dog on the left twice towards the right, then

2. Swapping the large dog on the right four times towards the left.

When swapping, every small dog must be involved in a swap since there is at least one small dog between two large dogs initially. Further, each swap must include a small dog **and** a large dog since swapping two small dogs is not useful. There are six small dogs in total between any two large dogs, so there must be at least six swaps. As shown above, achieving six swaps is possible, and trying to move all three dogs to the left or to the right instead of the middle requires more than six swaps.

Strawberry Hunt

Four beavers swim through canals in an attempt to find a strawberry. They start at different places and always move in the direction of the arrows shown below.



Each beaver either finds the strawberry, swims in a loop forever, or reaches and remains at a dead-end.

How many beavers find the strawberry?

A. 1

B. 2

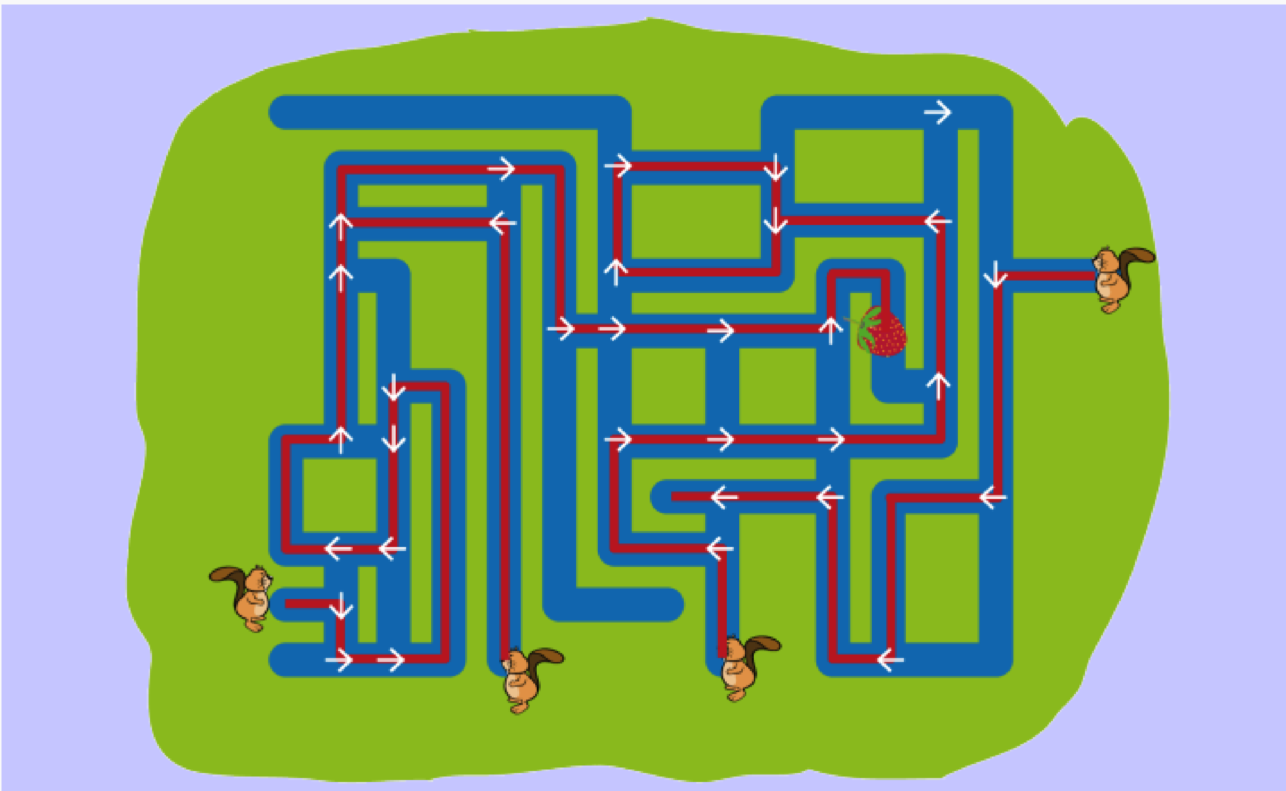
C. 3

D. 4

Solution

The answer is (B), two beavers find the strawberry.

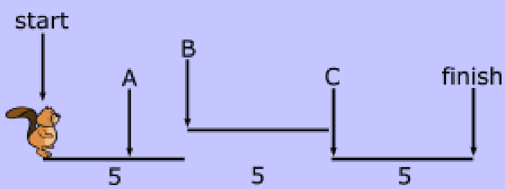
If we look at the paths of each of the beavers, we can see that it is the two beavers on the left that will reach the strawberry (indeed, their paths actually meet!). The third beaver at the bottom will start swimming in the circle, and the beaver on the very right swims into a dead end and has no way of getting out.



Jumpers

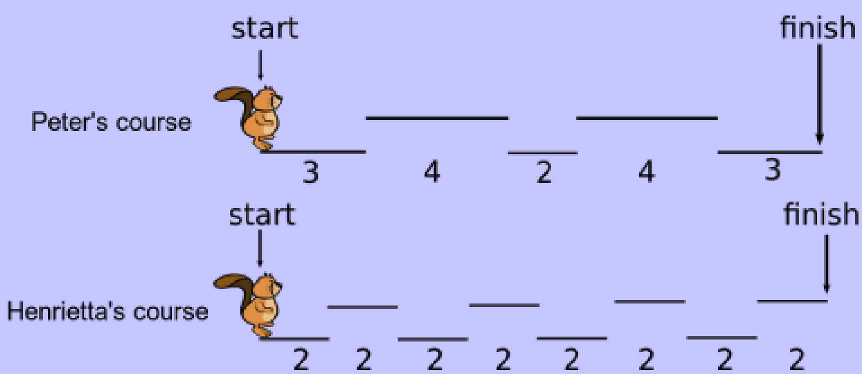
Peter and Henrietta are playing a video game. They move a beaver at a constant speed from the start of a course to the finish. The course consists of platforms on two levels. At the end of each platform before the finish, the beaver jumps instantaneously up or down to the next platform. The amount of time to move over each platform of the game is shown below each platform.

Here is an example course:



- 3 seconds after the start, the beaver is at A;
- 5 seconds after the start, the beaver is at B;
- 10 seconds after the start, the beaver is at C;
- 15 seconds after the start, the beaver is at the finish.

Peter and Henrietta start playing the following two different courses at exactly the same time.



For how long are both beavers moving along the top level at the same time?

A. 2 seconds

B. 4 seconds

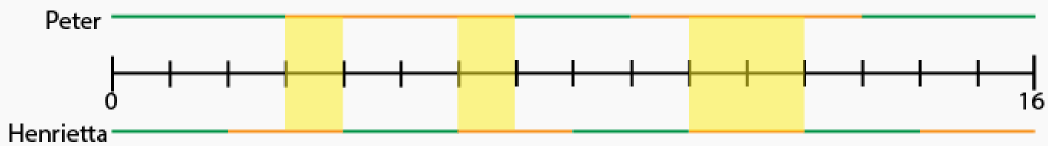
C. 6 seconds

D. 8 seconds

Solution

The answer is (B), both beavers are moving along the top level at the same time for a total of 4 seconds.

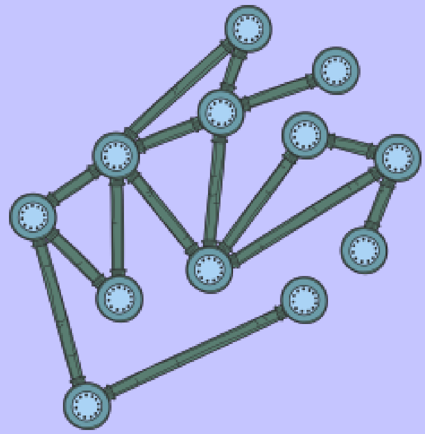
We can use a number line to show when Peter and Henrietta are moving along the bottom and top levels, and then look at when these times overlap:



If the green line represents moving along the bottom level and the orange line represents moving along the top level, we can see that there are three time slots where Peter and Henrietta's orange lines overlap. The first two times last 1 second each, and the third time lasts 2 seconds. So in total, there are 4 seconds when both beavers are moving along the top level at the same time.

Pipe Network

A network of 12 nodes connected by pipes is shown below. Exactly one node is clogged. However, even with this clog, water can flow between any pair of connected unclogged nodes in the network.



How many possibilities are there for the clogged node?

A. 5

B. 6

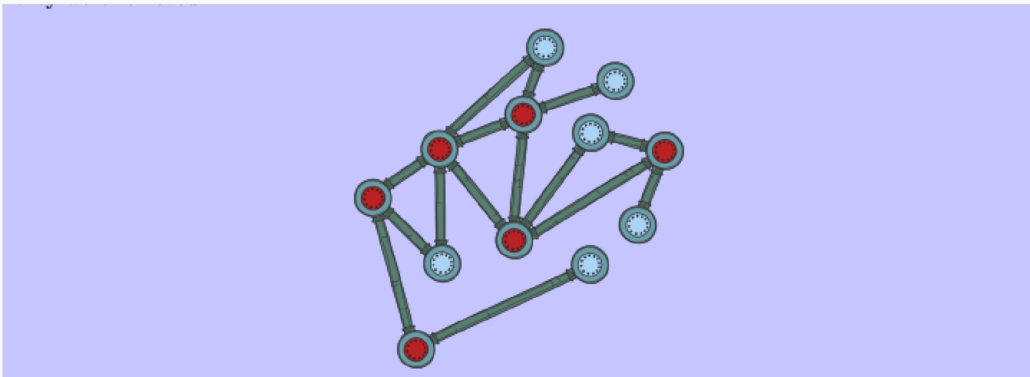
C. 7

D. 8

Solution

The answer is (B), there are six possibilities for the clogged node.

Looking at the diagram below, we can see that the six red nodes are the only ones which would disconnect the pipe network if they were removed. This leaves us with the other six nodes that would *not* disconnect the pipe network if they were removed.

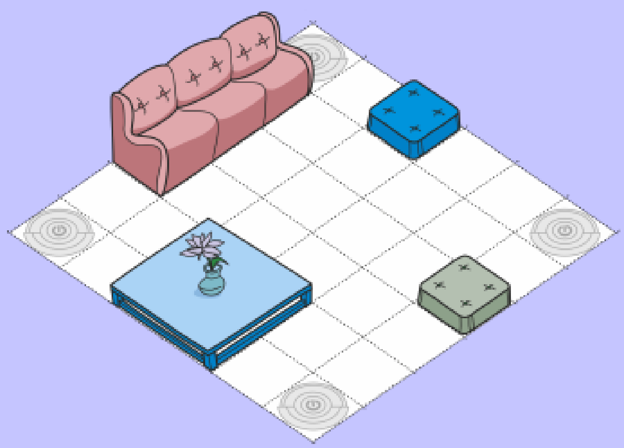


Robot Cleaner

A robot washes the square tiled floor shown below by using the following commands:

- F - move forward one tile (which takes 1 minute)
- W - wash a tile (which takes 1 minute)
- R - turn 90° right (which is performed instantly)
- L - turn 90° left (which is performed instantly)

The robot can start at any corner facing any direction and can end at any corner. It never goes on a tile occupied by one of the four pieces of furniture and washes all the other tiles, including the 4 corner tiles, exactly once. The robot may travel over a tile more than once.



What is the minimum possible number of minutes the robot needs to wash the entire floor?

A. 53

B. 54

C. 55

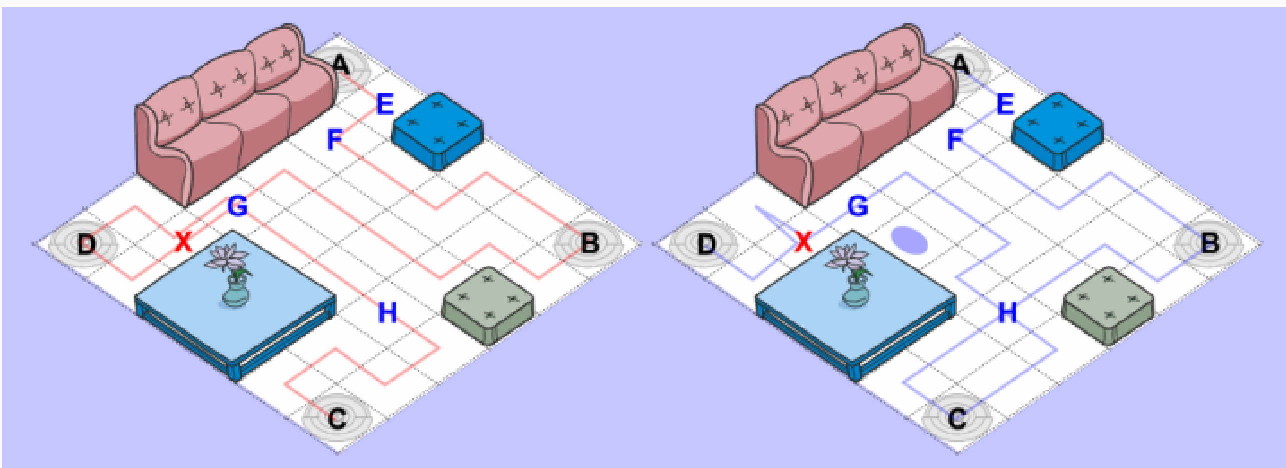
D. 56

Solution

The answer is (C), the robot needs a minimum of 55 minutes to wash the entire floor.

There are $36 - 9 = 27$ floor tiles, so washing takes 27 minutes. If every tile was visited once, moving would take 26 minutes (as we don't have to move to the first tile). So we have to minimize number of tiles which are passed twice.

Look at the pictures below. There is a tile X which must be passed twice every chosen way. Some of the tiles form a "narrow passage" to corner tiles A B C D: these tiles, labelled by the letters letters E, F, G, and H, must be passed over twice if the nearest corner to E, F, G, or H, is not a starting or ending tile. Tiles E and F are on the A narrow passage, tile G is on the D narrow passage, and H is on the C narrow passage. Corner B has no narrow passage tile. If robot starts or finishes at some corner, the narrow passage tile(s) on this corner can be passed through only once. Thus we can try to arrange the path so that robot went through as few narrow passage tiles as possible.



If the robot starts and finishes at the same corner tile it must pass all of the narrow passage tiles twice. We should thus make sure we start and end at different tiles to reduce our steps. As stated above, we want our robot to pass through as few narrow passage tiles as possible, and this includes trying to pass through them at most one time.Because corner A has two narrow passage tiles, E and F, a good strategy is to choose the corner A as a starting (or finishing) point. We then have to decide between finishing corners C and D (in both cases we spare another one narrow passage tiles of G or H).

Tracing the picture on the left shows that to move from A to C needs 28 moves forward (it contains points X and G). Tracing from A to D needs an odd number of moves so after using 28 moves forward at least one tile is not passed yet (in the right picture, one such tile which is not passed through yet is the one with the blue ellipse) so it is longer than from A to C.

Minimal time for cleaning is 27 (washing) + 28 (moving) minutes = 55 minutes.

About Gauss Mathematics Contests

The Gauss Contests introduce students in Grades 7 and 8 to a broader perspective of mathematics in a fun, accessible way. Intriguing problems and a multiple-choice format make the Gauss Contests a wonderful opportunity for all students in these grades to grow their interest in and get curious about the power of math.

Note

There is a Grade 7 Gauss contest and a Grade 8 Gauss contest. Although they might share some questions, they are different contests from each other. If you are in grade 7 or lower, you may choose to write either one of Gauss 7 or Gauss 8. If you are in grade 8, you **cannot** write Gauss 7.

Contest Information

Audience

- All students in Grades 7 and 8
- Interested students from lower grades

Contest Date

- Ordering (Registration) Deadline: April 25, 2023
- Contest Date: May 17, 2023 (North & South America)

Fees

- $5.00 per participant

Format

- Number of questions: 25 multiple-choice questions
- Duration: 60 minutes
- Score: out of 150
- Format (delivering) of the contest: paper or online
- Some calculators permitted
- Paper dictionaries allowed.

Calculating devices are allowed, provided that they do not have: internet access, the ability to communicate with other devices, information previously stored by students (such as formulas, programs, notes, etc.), a computer algebra system, dynamic geometry software.

Mathematical Content

Questions are based on curriculum common to all Canadian provinces. The last few questions are designed to test ingenuity and insight. Rather than testing content, most of the contest problems test logical thinking and mathematical problem-solving.

Recognition

- A Certificate of Participation is provided for each participant.
- A Certificate of Distinction is provided for each participant scoring in the top 25% of all participants within their own school, for schools with at least 4 participating students.
- A Certificate of Outstanding Achievement is provided to the highest achieving participant in their school on each of the Grade 7 and 8 Contests, for schools with at least 10 participating students.
- The names of some of the top-scoring participants among all those writing the contests are posted online.

Contest Supervisors have the option of generating and printing Participation, Distinction, and School Champion Certificates in our Contest Supervisor Portal.

Note
At the time of the contest, you will be given a package which contains BOTH Gauss 7 and Gauss 8. One side of the package will be Gauss 7 and the reverse side of the package will be Gauss 8. Please make sure to check that you are writing the correct one.

How to get the most marks out of what you know!

We will list options that will give you the most marks in order, according to the marking instructions:

1. A correct answer for Part C (8 marks)
2. A correct answer for Part B (6 marks)
3. A correct answer for Part A (5 marks)
4. An unanswered question (2 marks) up to 10 questions
5. An incorrect answer for any parts (0 marks)

It is logical that selecting the correct answer for as many questions as possible will gain you the most marks. But, what if you are not sure if the answer is correct or not?

- Fort Part A, a question is worth 5 marks. If you are choosing between 2 options, your probability of getting it right is $\frac{1}{2}$ and $\frac{1}{2} \times 5 = 2.5 > 2$. So you should guess. But if you are choosing between 3 options, you have $\frac{1}{3} \times 5 < 2$. So in this case you should not guess (leave it blank). Similarly, if you are choosing between 4 or 5 options, you should leave the question blank to earn the most marks.
- For Part B, if you are choosing between 4 or 5 options, you are better to leave it blank. If you are choosing between 3 options, you have $\frac{1}{3} \times 6 = 2$, so you can either guess or leave it blank. If you are choosing between 2 options, it is better to guess.
- For Part C, if you are choosing between all 5 options, it is better to leave it blank. If you are choosing between 4 options, either guess or leave it blank. If you are choosing between 2 or 3 options, it is better to guess.

Do not forget to count the number of unanswered questions, since only up to 10 unanswered questions would gain you marks!

Problem Solving Strategies

Some useful strategies when writing Gauss Contests:

1. **Using the information given.** When they give you an information, they are probably expecting you to use that information to solve the problem. Choosing which information to use and the order of using each piece information are also important when it comes to problem solving.
2. **Working from the 5 possible answers.** Remember, 1 out of 5 possible answers must be correct. You can look at the possible answers first, and rule out the ones that cannot be the correct answer.
3. **Drawing/Using the diagram.** Remember, diagrams are not drawn to scale. Hence, drawing your own diagram could be more helpful than using the one on the question. If the question does not provide a diagram, draw your own! Incorporating some information provided in the question on the diagram will be crucial to problem solving. For example, if they provide a length, indicate that on your diagram! Drawing some lines to further divide your diagram could help you in so many problems.
4. **Looking for patterns.** If they are asking you to find the 2023$^{\text{rd}}$ number in a list, they are NOT expecting you to write down all numbers that come before. Always look for patterns and think about ways to apply that pattern to find your answer!
5. **Working backward.** Since you are given 5 possible answers, you can always try them one by one and find the one that makes sense.

Past Contest Problems

1. The value of $4^2 - 2^3$ is

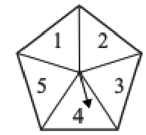
   (A) 8 (B) 2 (C) 4 (D) 0 (E) 6

   Solution

   ANSWER: (A) 8

   $4^2 - 2^3 = 16 - 8 = 8$

2. A pentagon is divided into 5 equal sections, as shown. An arrow is attached to the centre of the pentagon. The arrow is spun once. What is the probability that the arrow stops in the section numbered 4?

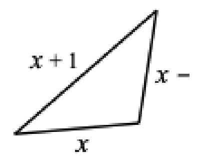


(A) $\frac{3}{5}$ (B) $\frac{1}{2}$ (C) $\frac{4}{5}$ (D) $\frac{1}{4}$ (E) $\frac{1}{5}$

Solution

ANSWER: (D) $\frac{1}{5}$

3. If the perimeter of the triangle shown is 21, what is the value of x?



(A) 3 (B) 7 (C) 8 (D) 13 (E) 16

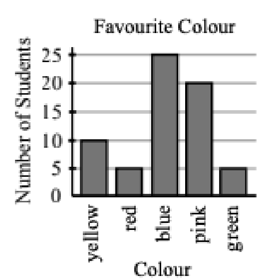
Solution

ANSWER: (B) 7

The perimeter of a shape is the sum of all its sides. This gives:

$$\begin{aligned}(x+1)+(x-1)+x &= 21\\ 3x &= 21\\ x &= 7\end{aligned}$$

4. Students were surveyed about their favourite colour and the results are displayed in the graph shown. What is the ratio of the number of students who chose pink to the number of students who chose blue?



(A) 4 : 5 (B) 3 : 5 (C) 1 : 5 (D) 2 : 5 (E) 5 : 3

Solution

ANSWER: (A) 4 : 5

If we look at the graph, we can see that 20 students chose pink and 25 students chose blue. Thus our ratio is 20 : 25, but we can reduce this further:

$20 \div 5 = 4$

$25 \div 5 = 5$

$\therefore\ 20 : 25 = 4 : 5$.

5. When a number is tripled and then decreased by 6, the result is 15. The number is

(A) 8 (B) 6 (C) 5 (D) 7 (E) 9

Solution

ANSWER: (D) 7

Let us call the number in question n and find an equivalent equation that represents the words described in the question:

- Tripled $\longrightarrow$ multiplication by 3 $\longrightarrow 3n$
- Decreased by 6 $\longrightarrow$ subtract 6 $\longrightarrow 3n - 6$
- Result is 15 $\longrightarrow$ result is equal to 15 $\longrightarrow 3n - 6 = 15$

We can now simply solve for n:

$$\begin{aligned} 3n - 6 &= 15 \\ 3n &= 15 + 6 \\ 3n &= 21 \\ n &= 21 \div 3 \\ n &= 7 \end{aligned}$$

6. The sum of the first 100 positive integers (that is, $1+2+3+\cdots+99+100$) equals 5050. The sum of the first 100 positive multiples of 10 (that is, $10+20+30+\cdots+990+1000$) equals

(A) 10 100 (B) 5950 (C) 50 500 (D) 6050 (E) 45 450

Solution

ANSWER: (C) 50 500

Notice that in $10+20+30+\cdots+990+1000$, each term is 10 times larger than its corresponding term in the sum $1+2+3+\cdots+99+100$. It then follows that the required sum is 10 times larger than the sum that has been given:

Since $1+2+3+\cdots+99+100 = 5050$, $10+20+30+\cdots+990+1000 = 5050 \times 10 = 50\ 500$.

7. Brodie and Ryan are driving directly towards each other. Brodie is driving at a constant speed of 50 km/h. Ryan is driving at a constant speed of 40 km/h. If they are 120 km apart, how long will it take before they meet?

(A) 1h 12min (B) 1h 25min (C) 1h 15min (D) 1h 33min (E) 1h 20min

Solution

ANSWER: (E) 1h 20min

When Brodie and Ryan are driving directly towards each other at their constant speeds of 50km/h and 40km/h, respectively, then the distance between them is decreasing at a rate of 90km/h ($50 + 40 = 90$).

If the initial distance between Brodie and Ryan is 120km, and the distance is decreasing at 90km/h, then they will meet after $\frac{120}{90}$ hours, or $1\frac{1}{3}$ hours.

$\frac{1}{3}$ of an hour is $\frac{1}{3} \times 60 = 20$ minutes, so it will take Brodie and Ryan 1 h 20 min to meet.

8. In a group of seven friends, the mean (average) age of three of the friends is 12 years and 3 months and the mean age of the remaining four friends is 13 years and 5 months. In months, the mean age of all seven friends is

(A) 156 (B) 154 (C) $155\frac{1}{2}$ (D) 157 (E) 155

Solution

ANSWER: (E) 155

If the average age of three of the friends is 147 months ($12 \times 12 + 3 = 147$) then the sum of their ages is $147 \times 3 = 441$ months.

If the average age of the other four friends is 161 months ($13 \times 12 + 5 = 161$) then the sum of their ages is $161 \times 4 = 644$ months.

The total sum of the ages of the seven friends is $441 + 644 = 1085$, and so the mean age of all seven friends is $1085 \div 7 = 155$ months.

9. Brady is stacking 600 plates in a single stack. Each plate is coloured black, gold or red. Any black plates are always stacked below any gold plates, which are always stacked below any red plates. The total number of black plates is always a multiple of two, the total number of gold plates is always a multiple of three, and the total number of red plates is always a multiple of six. For example, the plates could be stacked with:

- 180 black plates below 300 gold plates below 120 red plates, or
- 450 black plates below 150 red plates, or
- 600 gold plates.

In how many different ways could Brady stack the plates?

(A) 5139 (B) 5142 (C) 5145 (D) 5148 (E) 5151

Solution

ANSWER: (E) 5151

There are different ways that you could go about solving this question; we will be following the solution below:

In a given way of stacking the plates, let b be the number of groups of 2 black plates, g be the number of groups of 3 gold plates, and r be the number of groups of 6 red plates. Then there are $2b$ black plates, $3g$ gold plates, and $6r$ red plates.

Since the total number of plates in a stack is 600, we have $2b + 3g + 6r = 600$.

We note that the numbers of black, gold and red plates completely determines the stack (we cannot rearrange the plates in any way), and so the number of ways of stacking the plates is the same as the number of ways of solving the equation $2b + 3g + 6r = 600$ where b, g, r are integers that are greater than or equal to 0.
Since r is at least 0 and $6r$ is at most 600, then the possible values for r are 0, 1, 2, 3, . . . , 98, 99, 100.
When $r = 0$, we obtain $2b + 3g = 600$.
Since g is at least 0 and $3g$ is at most 600, g is at most 200.
Because $2b$ and 600 are even, $3g$ is even, and so g is even.
Therefore, the possible values for g are 0, 2, 4, . . . , 196, 198, 200.
Since $200 = 100 \times 2$, there are 101 possible values for g.
When $g = 0$, we get $2b = 600$ and so $b = 300$.
When $g = 2$, we get $2b = 600 - 3 \times 2 = 594$ and so $b = 297$.
Each time we increase g by 2, the number of gold plates increases by 6, so the number of black plates must decrease by 6, and so b decreases by 3.
Thus, as we continue to increase g by 2s from 2 to 200, the values of b will decrease by 3s from 297 to 0.
In other words, every even value for g does give an integer value for b.
Therefore, when $r = 0$, there are 101 solutions to the equation.
When $r = 1$, we obtain $2b + 3g = 600 - 6 \times 1 = 594$.
Again, g is at least 0, is even, and is at most $594 \div 3 = 198$.
Therefore, the possible values of g are 0, 2, 4, . . . , 194, 196, 198.
Again, each value of g gives a corresponding integer value of b.
Therefore, when $r = 1$, there are 100 solutions to the equation.
Consider the case of an unknown value of r, which gives $2b + 3g = 600 - 6r$.
Again, g is at least 0 and is even.
Also, the maximum possible value of g is $\frac{600-6r}{3} = 200 - 2r$.
This means that there are $(100 - r) + 1 = 101 - r$ possible values for g. (Can you see why?)
Again, each value of g gives a corresponding integer value of b.
Therefore, for a general r between 0 and 100, inclusive, there are $101 - r$ solutions to the equation.
We make a table to summarize the possibilities:

r	g	b	# of solutions
0	0, 2, 4, . . ., 196, 198, 200	300, 297, 294, . . ., 6, 3, 0	101
1	0, 2, 4, . . ., 194, 196, 198	297, 294, 291, . . ., 6, 3, 0	100
2	0, 2, 4, . . ., 192, 194, 196	294, 291, 288, . . ., 6, 3, 0	99
⋮	⋮	⋮	⋮
98	0, 2, 4	6, 3, 0	3
99	0, 2	3, 0	2
98	0	0	1

Therefore, the total number of ways of stacking the plates is

$$101 + 100 + 99 + ... + 3 + 2 + 1$$

We note that the integers from 1 to 100 can be grouped into 50 pairs each of which has a sum of 101 ($1 + 100$, $2 + 99$, etc) $\therefore$ the number of ways that Brady could stack the plates is $101 + 50 \times 101 = 5151$.